Determining the Earth’s gravity field using space-borne clocks

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Our Earth’s gravity field

• Central gravitational field: \( V = \frac{GM}{r} \)

• Spherical harmonic expansion: 

\[
V = \frac{GM}{r} + \frac{GM}{R} \sum_{n=2}^{N} \left( \frac{R}{r} \right)^{n+1} \sum_{m=0}^{n} \left[ C_{nm} \cos(m\lambda) + S_{nm} \sin(m\lambda) \right] P_{nm}(\cos\theta)
\]

Number of parameters: \( \approx N^2 \)

• Spatial resolution: \( D = \frac{200000}{N} \) km

\( N = 10 \)  \quad \( N = 50 \)  \quad \( N = 100 \)  \quad \( N = 200 \)
Methods to observe the global gravity field

Gravity field measurements

• gravity potential \((V)\)
  \[ V_i = \nabla V = \frac{\partial V}{\partial x_i} \]

• gravity accelerations \((V_i = \nabla V = \frac{\partial V}{\partial x_i})\)
  \[ V_{ij} = \nabla^2 V = \frac{\partial^2 V}{\partial x_i \partial x_j} \]

Credit: R. Rummel (1997)
Relativistic geodesy with clocks

\[ \frac{\Delta f}{f} \approx \frac{W_1 - W_2}{c^2} + O(c^{-4}) \]

\[ W = V + Z \]

Clocks can provide two kinds of important measurements in geodesy:

- gravity potential differences
- physical height differences between distant sites
Space-to-ground clock comparison

- Ground/reference clocks
- Clock on LEO satellite

Potential value $V$
Simulation scheme

Noise definition → Noise generator → Noisy observations \((V, \Delta V)\) → Signal synthesis → \(l = Ax\) → Gravity field coefficients

Comparison → LS adjustment

Input orbit

Reference gravity field model
Signal synthesis:

- Reference model: EIGEN-6c4, d/o 180
- Orbit: GRACE, 5 s

Noise simulation:

- Orbit error: 1.0, 1.0, 1.4 cm in X, Y, Z directions
- AOD error: AOD RL5 and RL6, d/o 100
- Clock error: white noise with different magnitudes (10^{-16} \sim 10^{-19})

Recovered gravity field models:

- Monthly solutions up to d/o 60 and 80
Recovered solutions

Degree-error RMS in geoid height

Clock error only

One-month GRACE satellite A orbit (January 2006) @ ~475 km
Recovered solutions

Degree-error RMS in geoid height

Clock error and AOD error

One-month GRACE satellite A orbit (January 2006) @ ~475 km
Recovered solutions

Signal-to-noise ratio: \( \log_{10}\left| \frac{\tilde{C}_{nm} \tilde{S}_{nm}}{\tilde{C}_{nm} \tilde{S}_{nm}} \right| \)

Clock noise: \( 1.0 \times 10^{-18} \)
Space-to-space clock comparison

Clocks on two LEO satellites

Potential difference $\Delta V$
One-month GRACE satellite A & B orbit (January 2006) @ ~475 km
Solutions from differential measurements in space

Degree-error RMS in geoid height

Clock error and AOD (RL6 – RL5)

One-month GRACE satellite A & B orbit (January 2006)
@ ~475 km
Combined scenario

- Ground/reference clocks
- Clocks on two LEO satellites

Potential $V$ and potential differences $\Delta V$
Solutions for the combined scenario

Degree-error RMS in geoid height

Clock error \((10^{-18})\)

One-month GRACE satellite A & B orbit
(January 2006)
@ ~475 km
Clocks for other geodetic applications?
Height system unification

Clocks are powerful in obtaining height differences between distant points. This makes them appropriate for height system unification, by identifying:

- discrepancies (offsets) between different height datums;
- systematic distortions of national/regional levelling networks.
Mass loss in Greenland

Clocks can detect the mass loss in some areas. Being complementary to GRACE, clocks provide:
• point-wise and
• high-frequency sampling obs.
Space geodetic reference frame

Variations of gravity potential at different altitudes

Altitude = 0 km, $\sigma = 282.57 \text{ m}^2/\text{s}^2$
Gravity field above the equator at different altitudes

Potential: [m²/s²]

σ₁ = 388.7
σ₂ = 292.7
σ₃ = 3.5
σ₄ = 0.8

0 km  450 km  20000 km  35786 km
Clocks in higher orbits support realizing a global gravity or height reference system, which is:

• stable/robust over time;
• easy to maintain.
Summary

- For gravity field determination, clocks:
  - deliver the gravity potential (difference), which is a scalar quantity and robust to attitude errors;
  - are sensitive to low-degree gravity field signals;
  - can detect the temporal signal below d/o 12 if uncertainty <10^{-18}

- As further geodetic applications, clocks can:
  - unify local height systems;
  - monitor mass changes like in Greenland;
  - realize a global gravity/height reference system;
  - ...

Open issues for future work
Doppler effects, configuration of clock networks, procedure for frequency comparison, long-term stability of clocks, ...