Determining the Earth's gravity field using space-borne clocks

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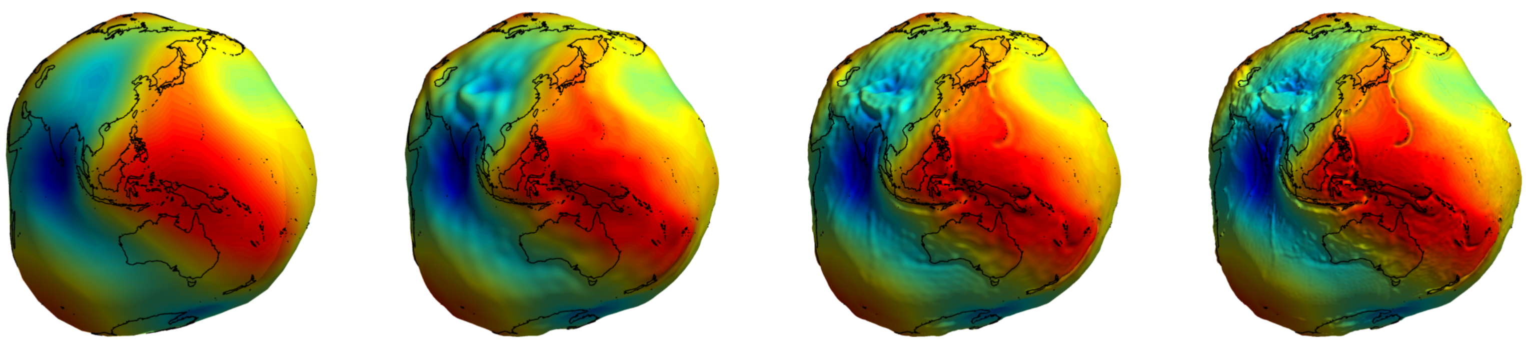
Our Earth's gravity field

- Central gravitational field: $V = \frac{GM}{r}$
- Spherical harmonic expansion:

Number of parameters: $\approx N^2$

$$V = \frac{GM}{r} + \frac{GM}{R} \sum_{n=2}^{N} \left(\frac{R}{r}\right)^{n+1} \sum_{m=0}^{n} \left[\bar{C}_{nm}\cos(m\lambda) + \bar{S}_{nm}\sin(m\lambda)\right] \bar{P}_{nm}(\cos\theta)$$

- Spatial resolution: $D = \frac{20000}{N}$ km

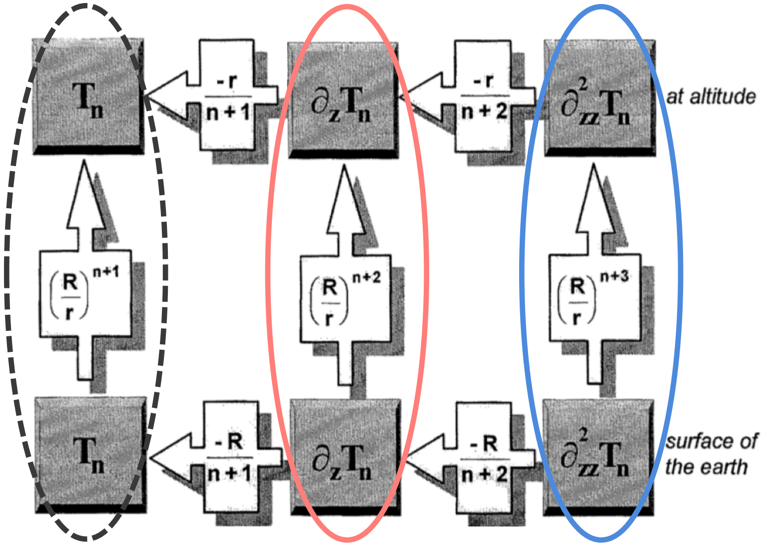


$N = 10$ $N = 50$ $N = 100$ $N = 200$

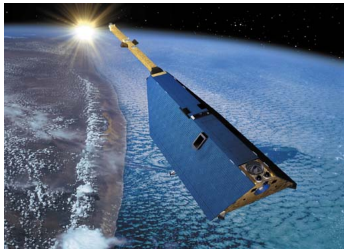
Methods to observe the global gravity field

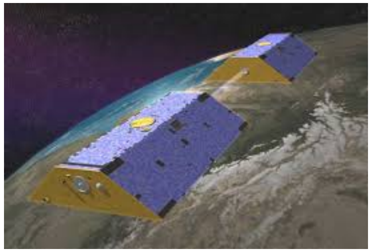
Gravity field measurements

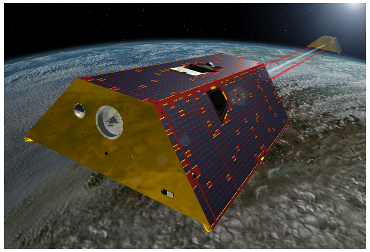
- gravity potential (V)
- gravity accelerations ($V_i = \nabla V = \frac{\partial V}{\partial x_i}$)
- gravity gradients ($V_{ij} = \nabla^2 V = \frac{\partial^2 V}{\partial x_i \, \partial x_j}$)



Credit: R. Rummel (1997)



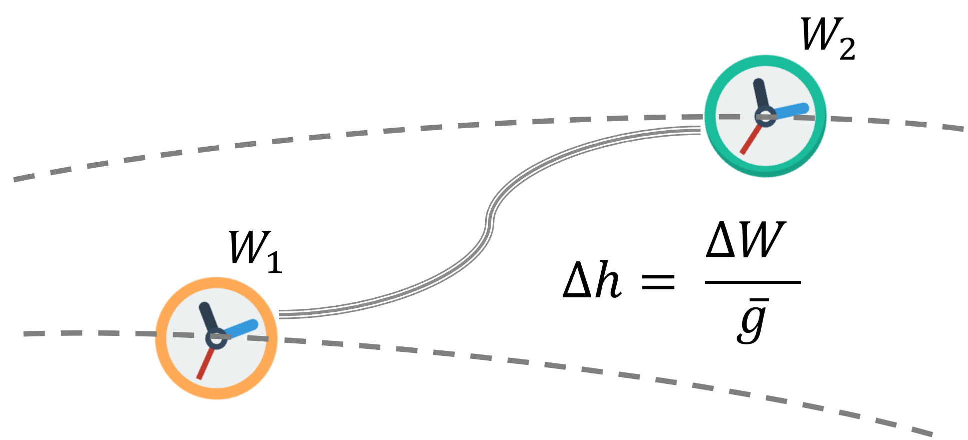




Relativistic geodesy with clocks

$$\frac{\Delta f}{f} \approx \frac{W_1 - W_2}{c^2} + O(c^{-4})$$

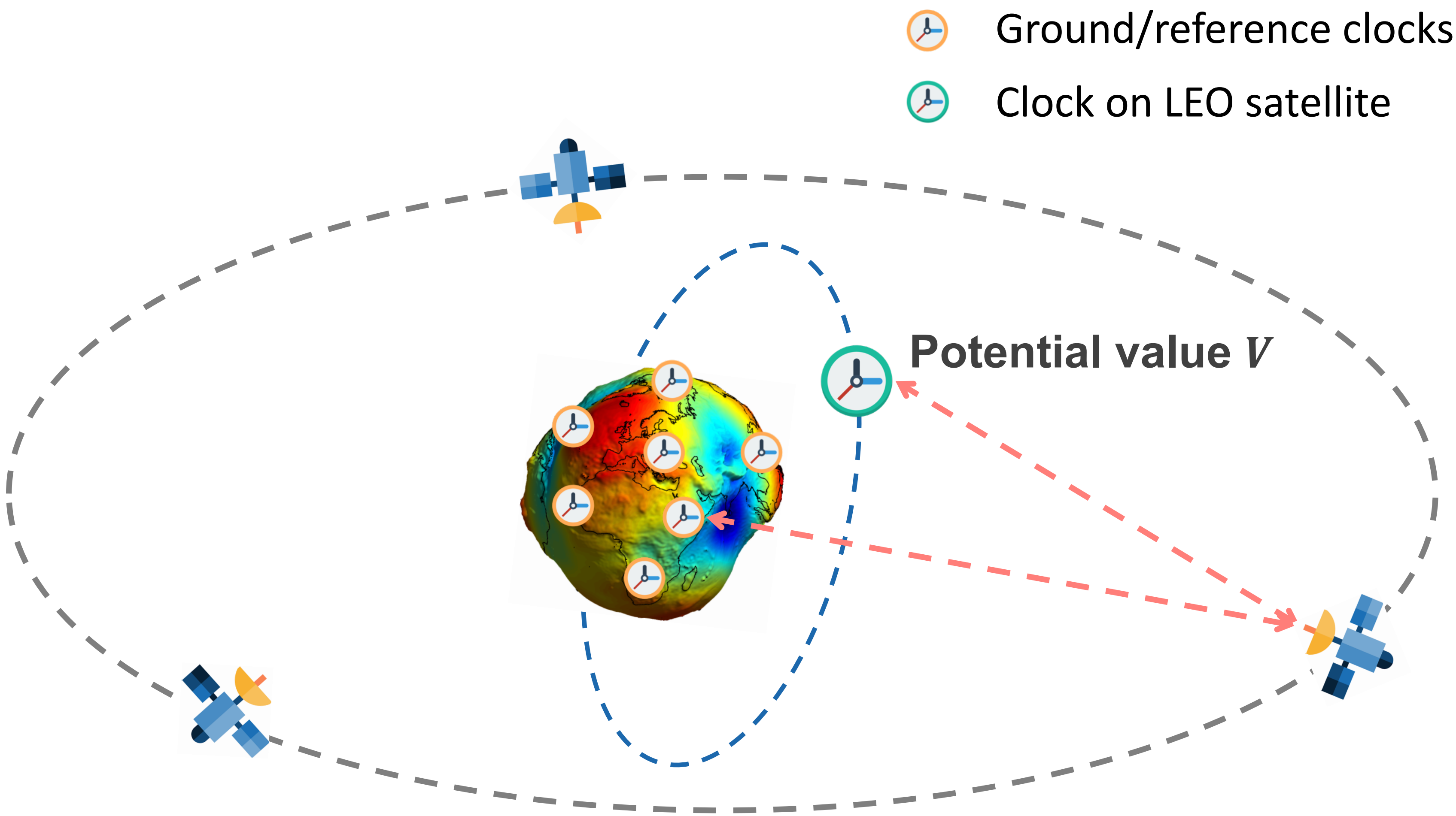
$$W = V + Z$$



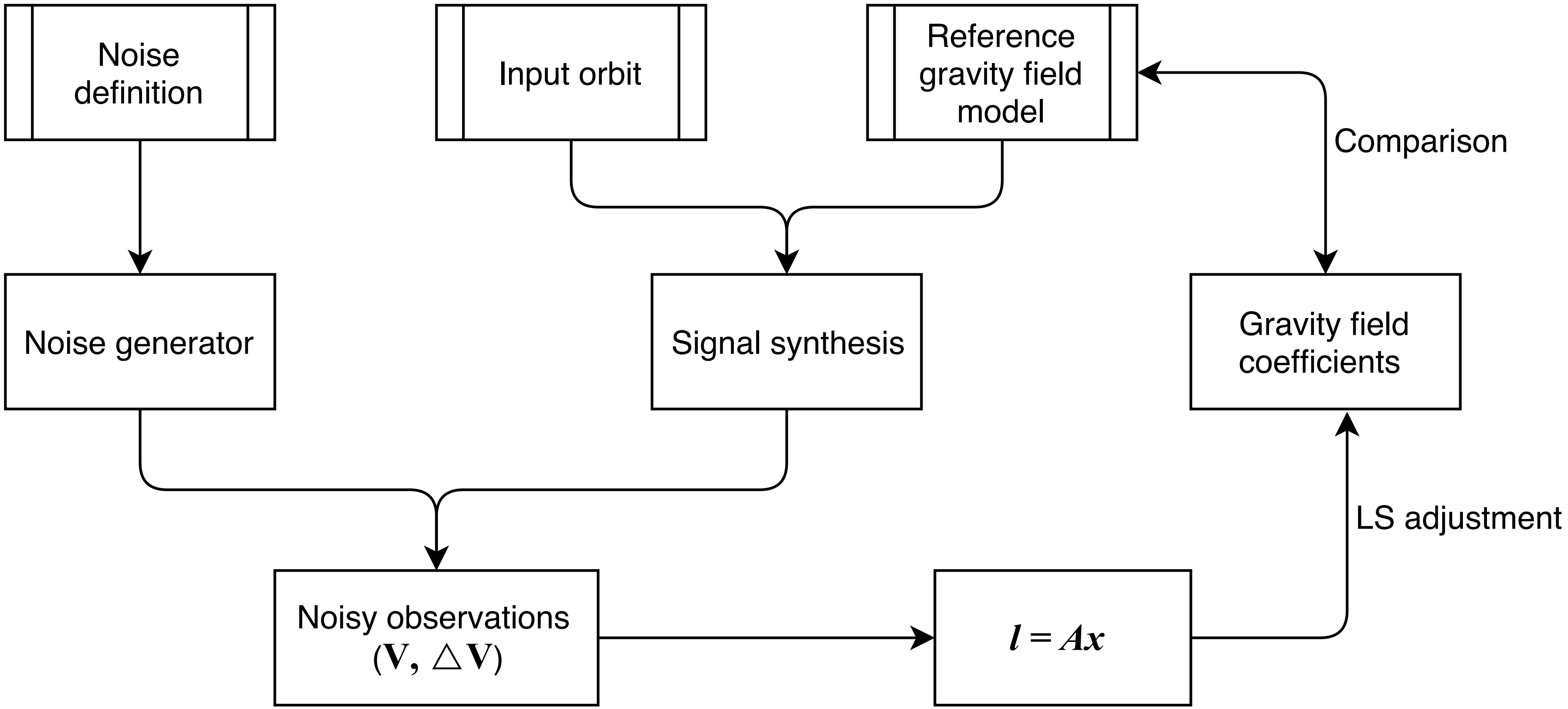
Clocks can provide two kinds of important measurements in geodesy

- gravity potential differences
- physical height differences between distant sites

Space-to-ground clock comparison



Simulation scheme



Data input and output

Signal synthesis:

- Reference model: EIGEN-6c4, d/o 180
- Orbit: GRACE, 5 s

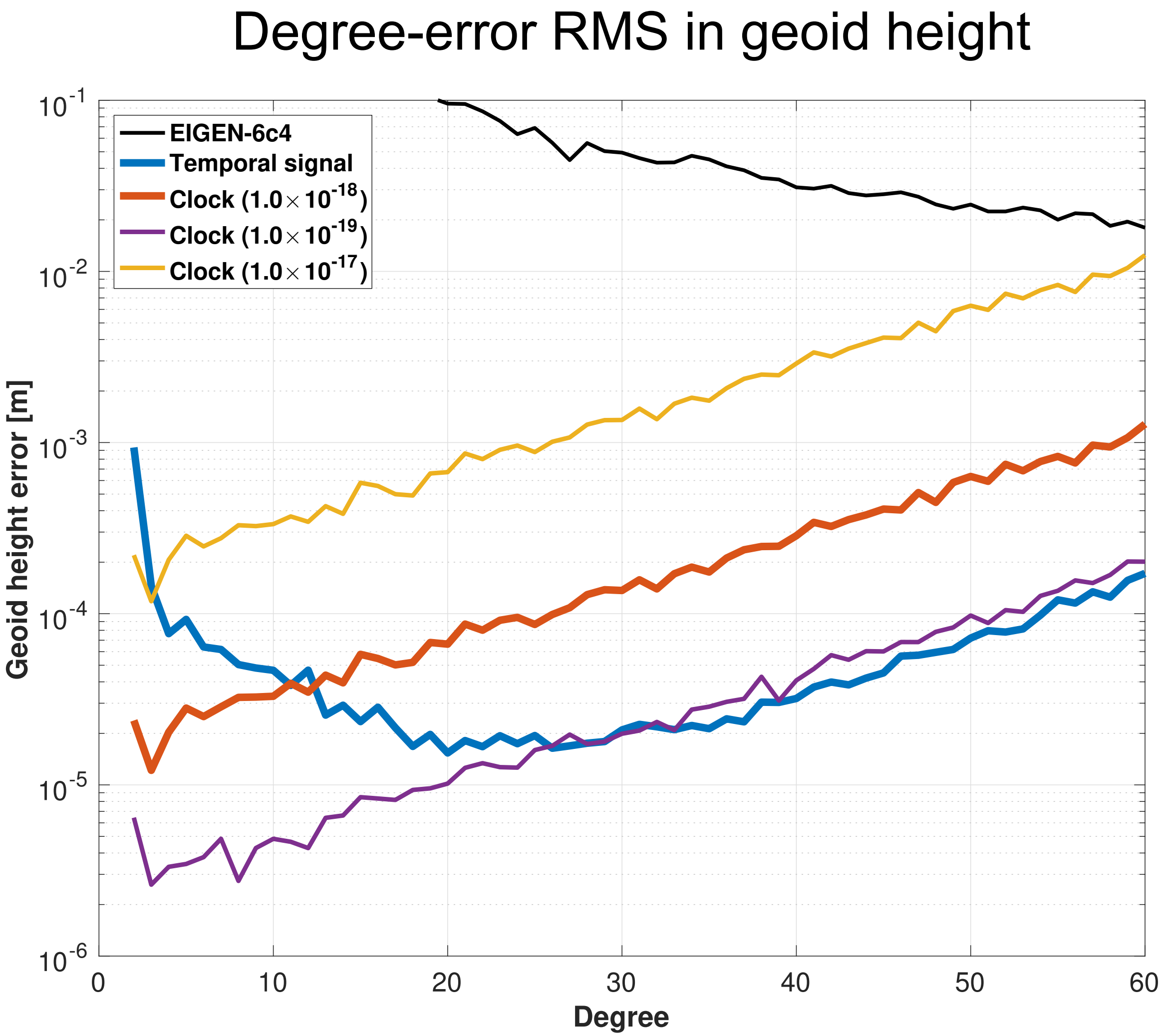
Noise simulation:

- Orbit error: 1.0, 1.0, 1.4 cm in X, Y, Z directions
- AOD error: AOD RL5 and RL6, d/o 100
- Clock error: white noise with different magnitudes ($10^{-16} \sim 10^{-19}$)

Recovered gravity field models:

- Monthly solutions up to d/o 60 and 80

Recovered solutions



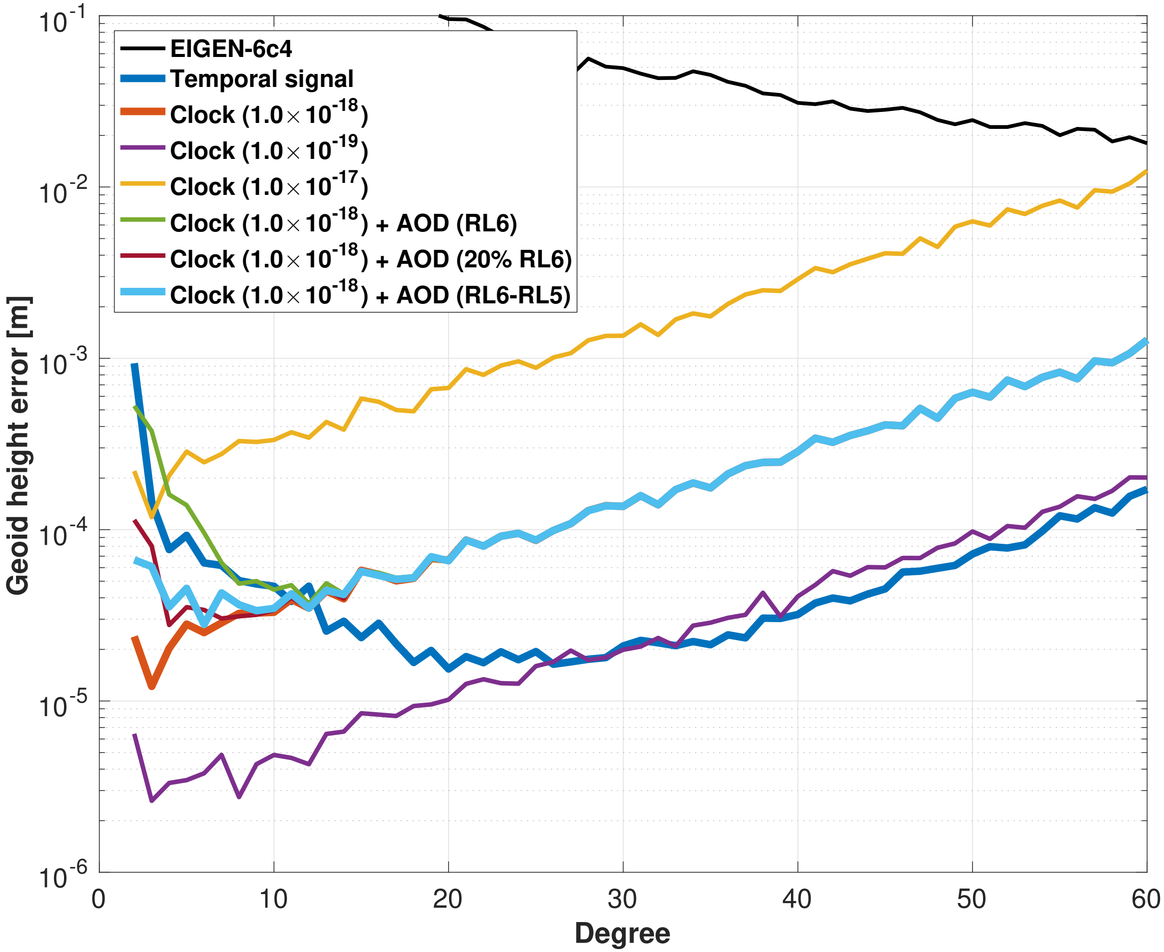
Clock error only

One-month GRACE satellite A orbit (January 2006)

@ ~475 km

Recovered solutions

Degree-error RMS in geoid height



Clock error and AOD error

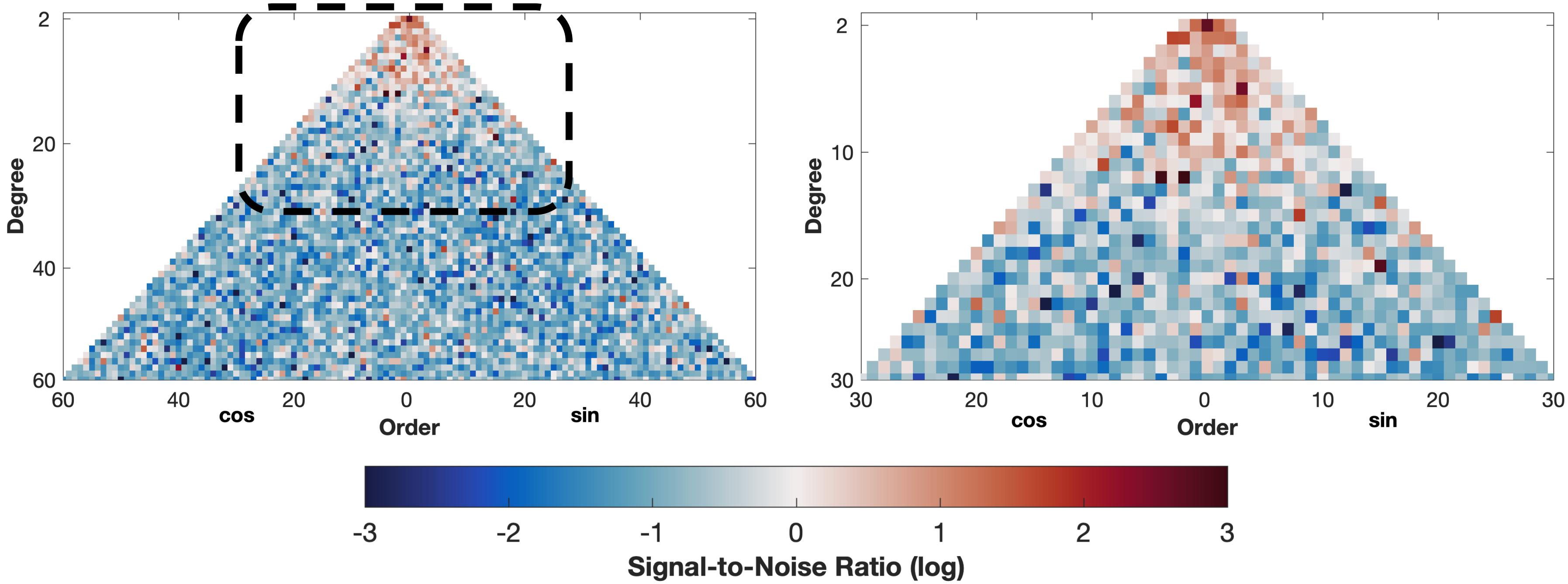
One-month GRACE satellite A orbit (January 2006)

@ ~475 km

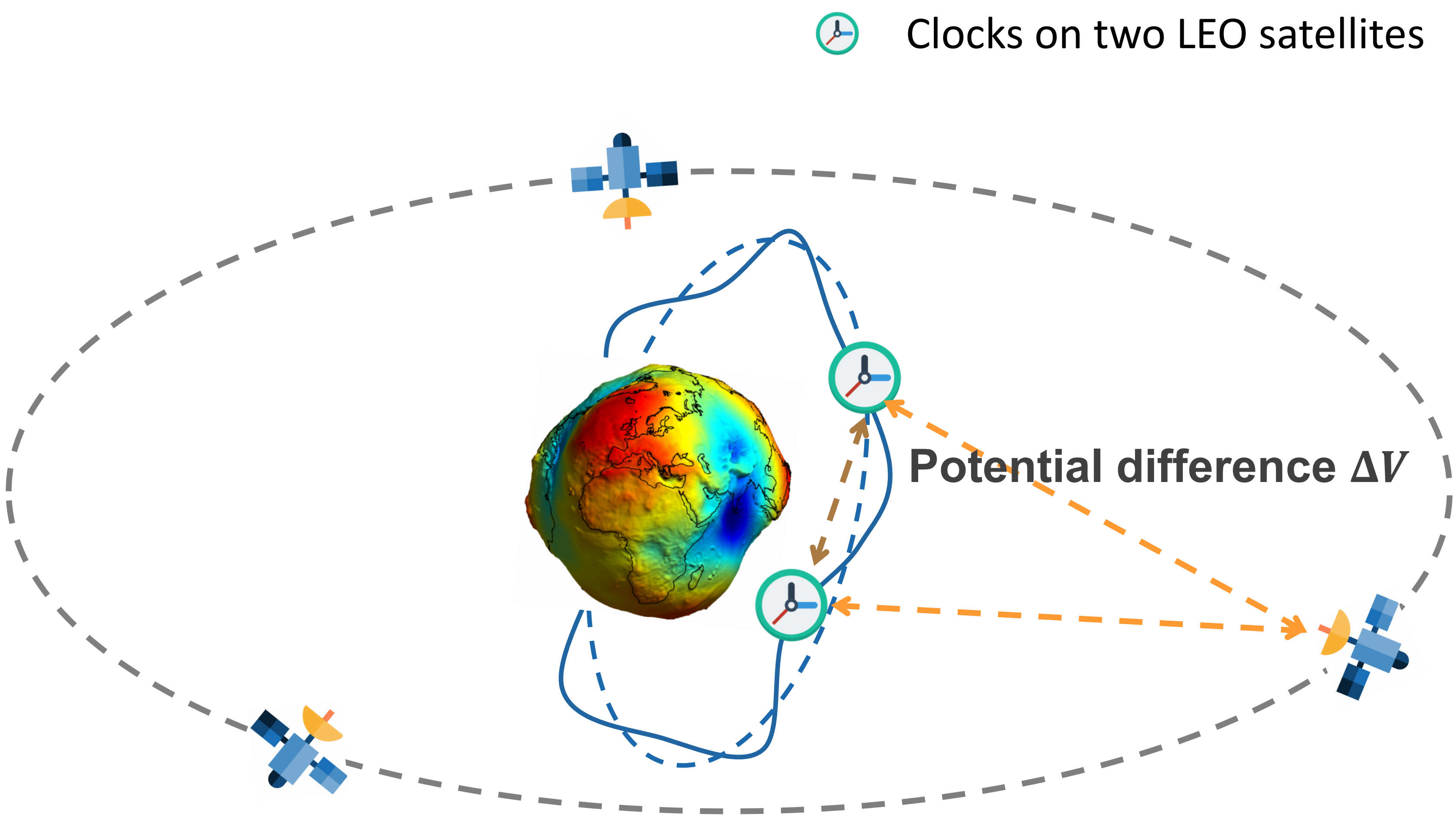
Recovered solutions

Signal-to-noise ratio: $\log_{10}\left|\frac{\{\bar{C}^{s}_{nm}, \bar{S}^{s}_{nm}\}}{\{\bar{C}^{n}_{nm}, \bar{S}^{n}_{nm}\}}\right|$

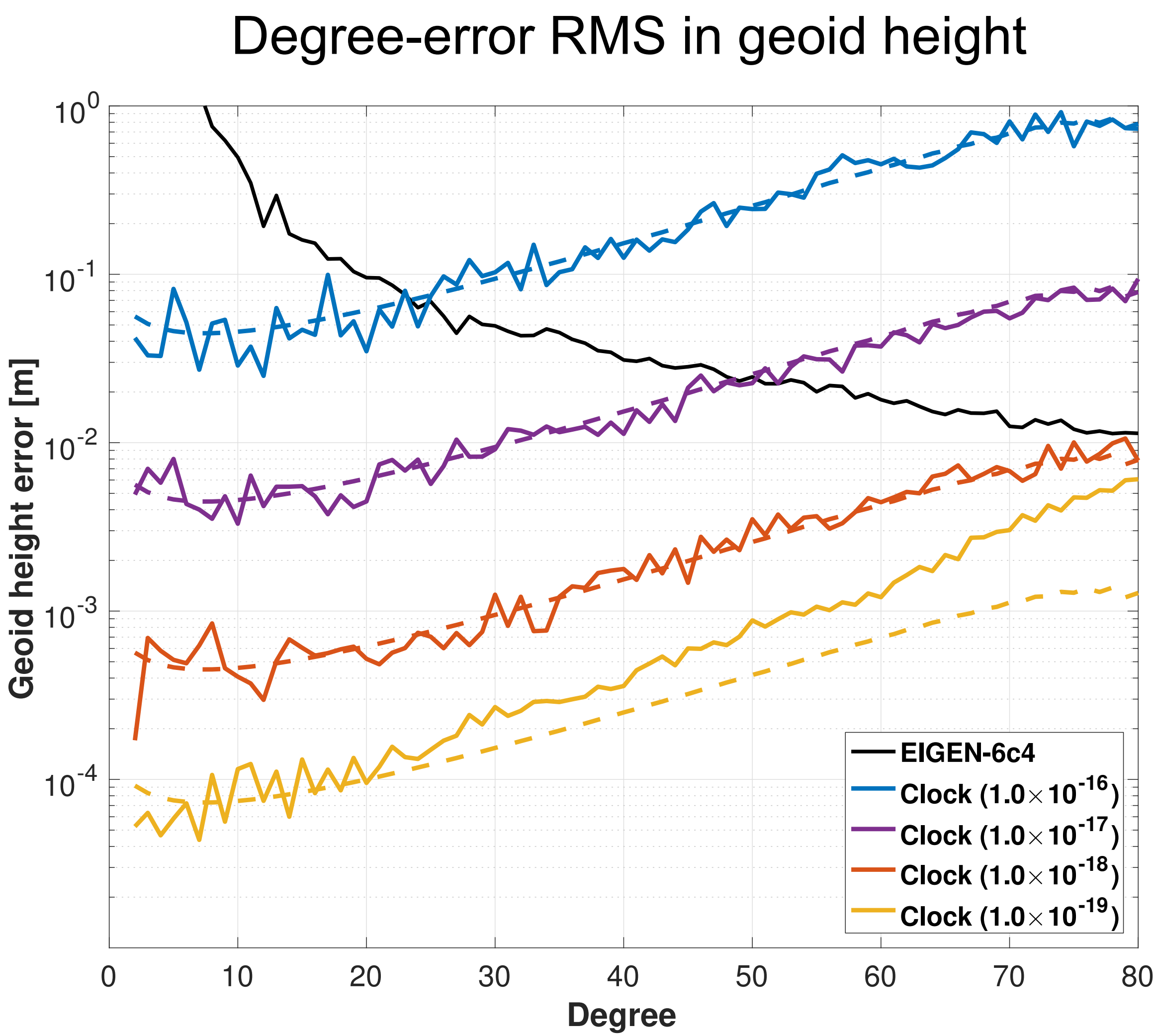
Clock noise: 1.0×10^{-18}



Space-to-space clock comparison



Solutions from differential measurements in space

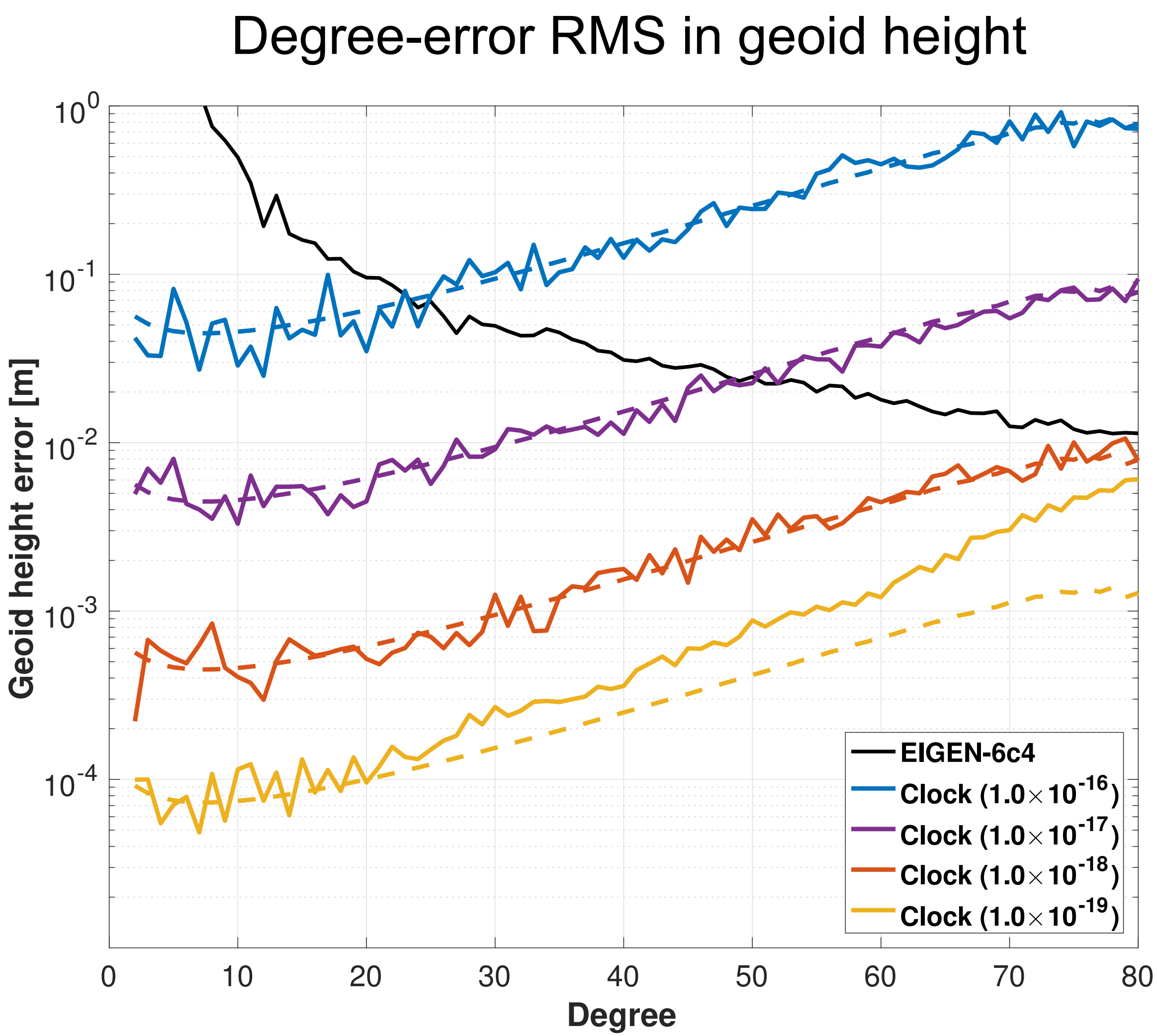


Clock error only

One-month GRACE satellite A & B orbit (January 2006)

@ ~475 km

Solutions from differential measurements in space

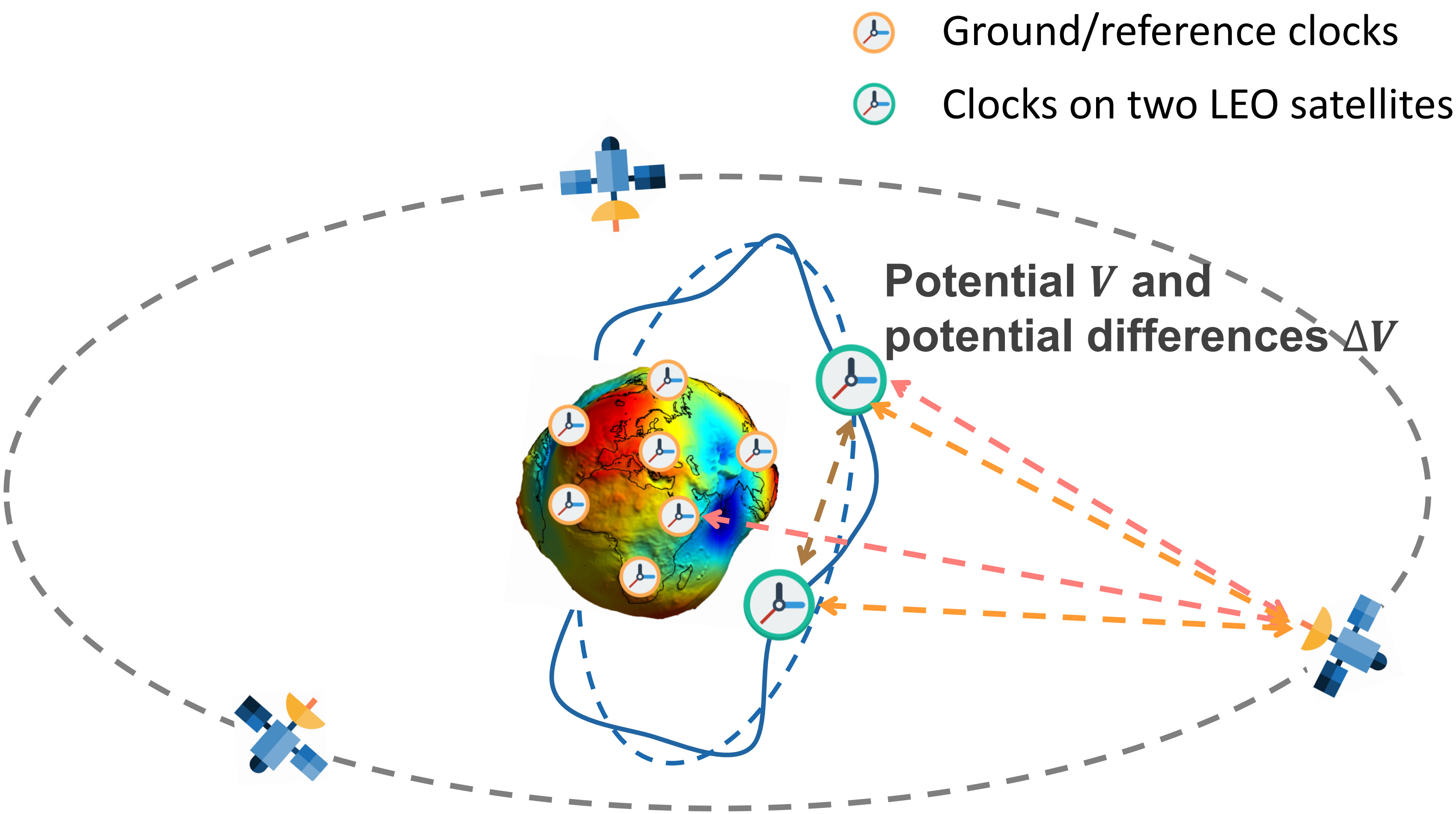


Clock error and AOD (RL6 – RL5)

One-month GRACE satellite A & B orbit (January 2006)

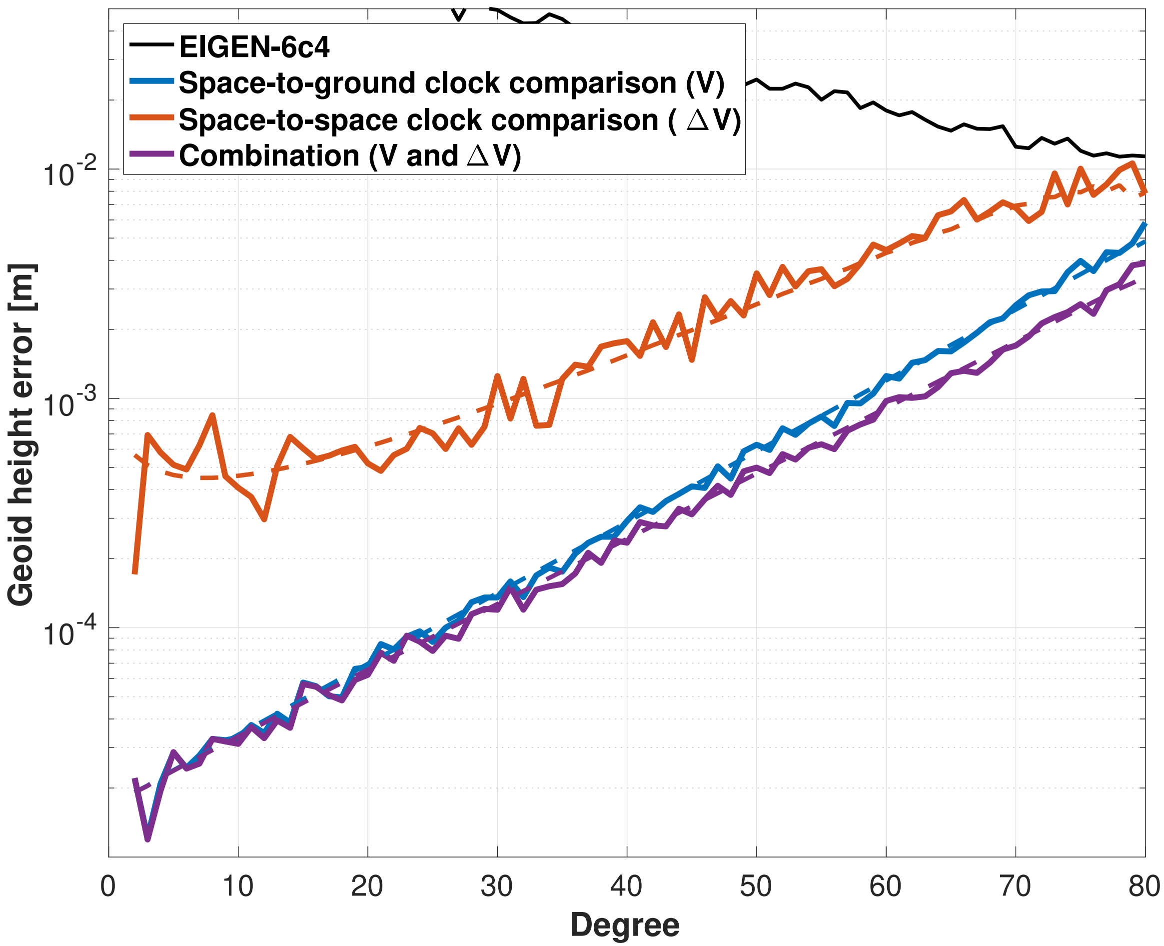
@ ~475 km

Combined scenario



Solutions for the combined scenario

Degree-error RMS in geoid height



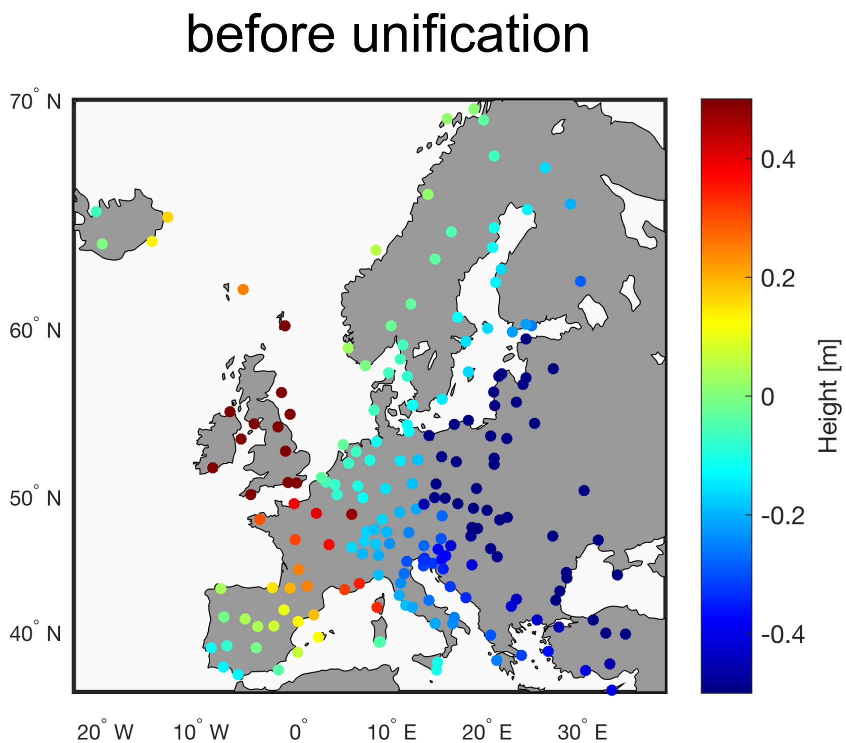
Clock error (10^{-18})

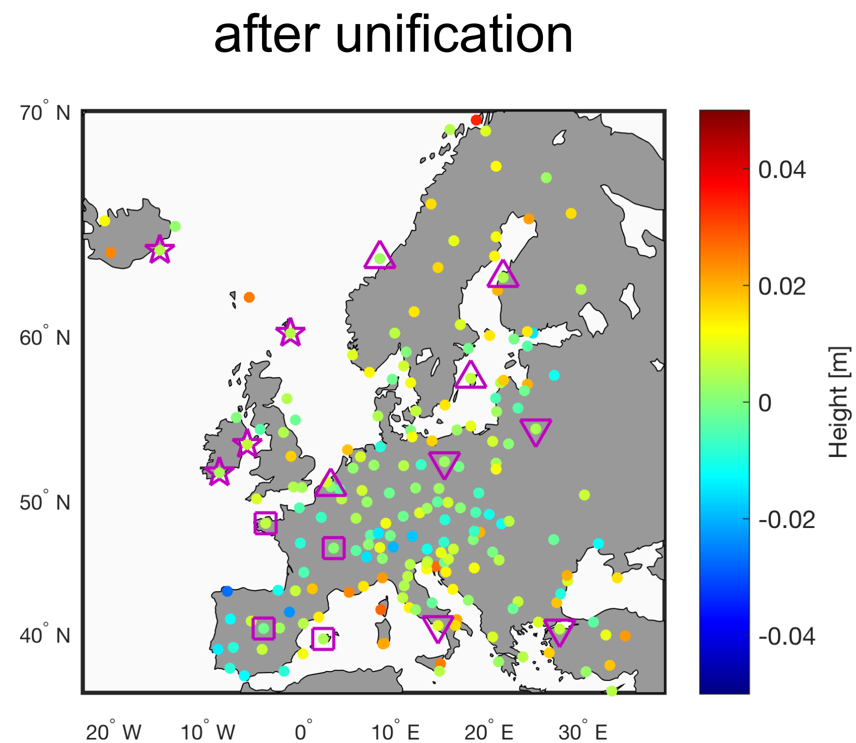
One-month GRACE satellite A & B orbit (January 2006)

@ ~475 km

Clocks for other geodetic applications?

Height system unification

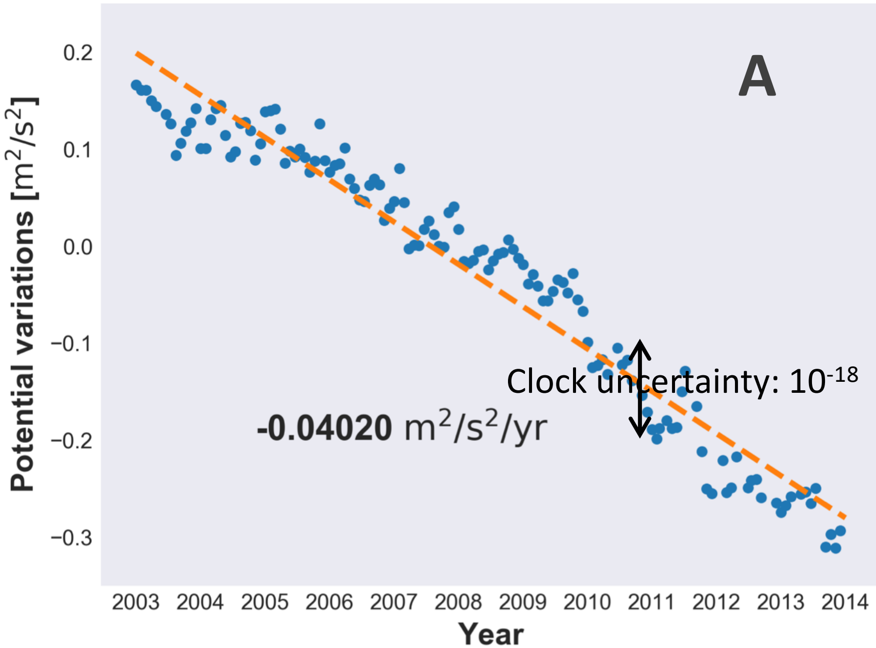


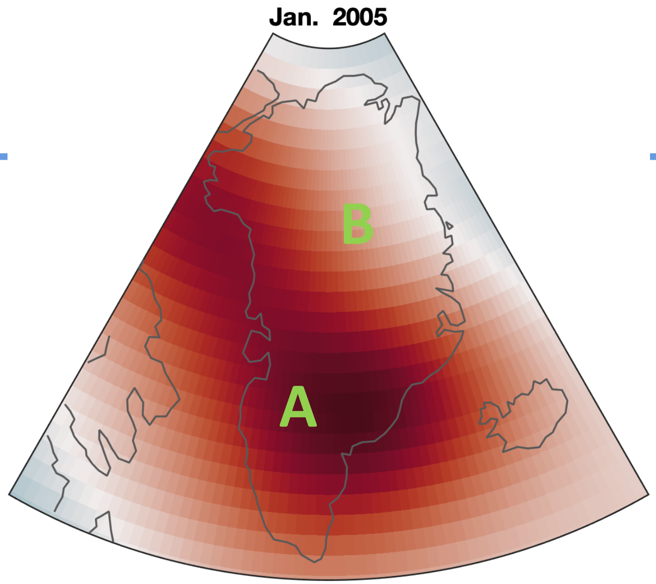


Clocks are powerful in obtaining height differences between distant points. This makes them appropriate for height system unification, by identifying:

- discrepancies (offsets) between different height datums;
- systematic distortions of national/regional levelling networks.

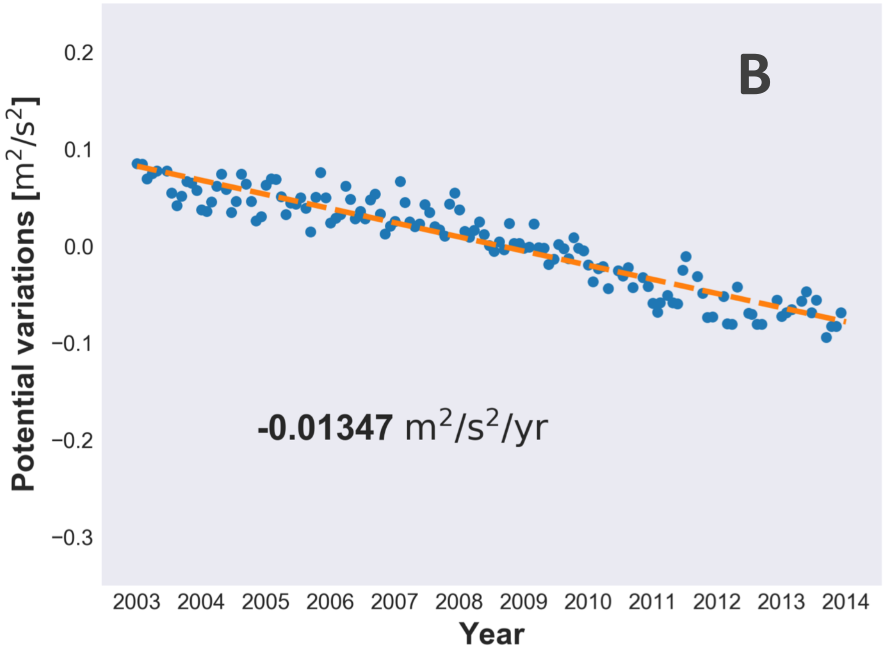
Mass loss in Greenland





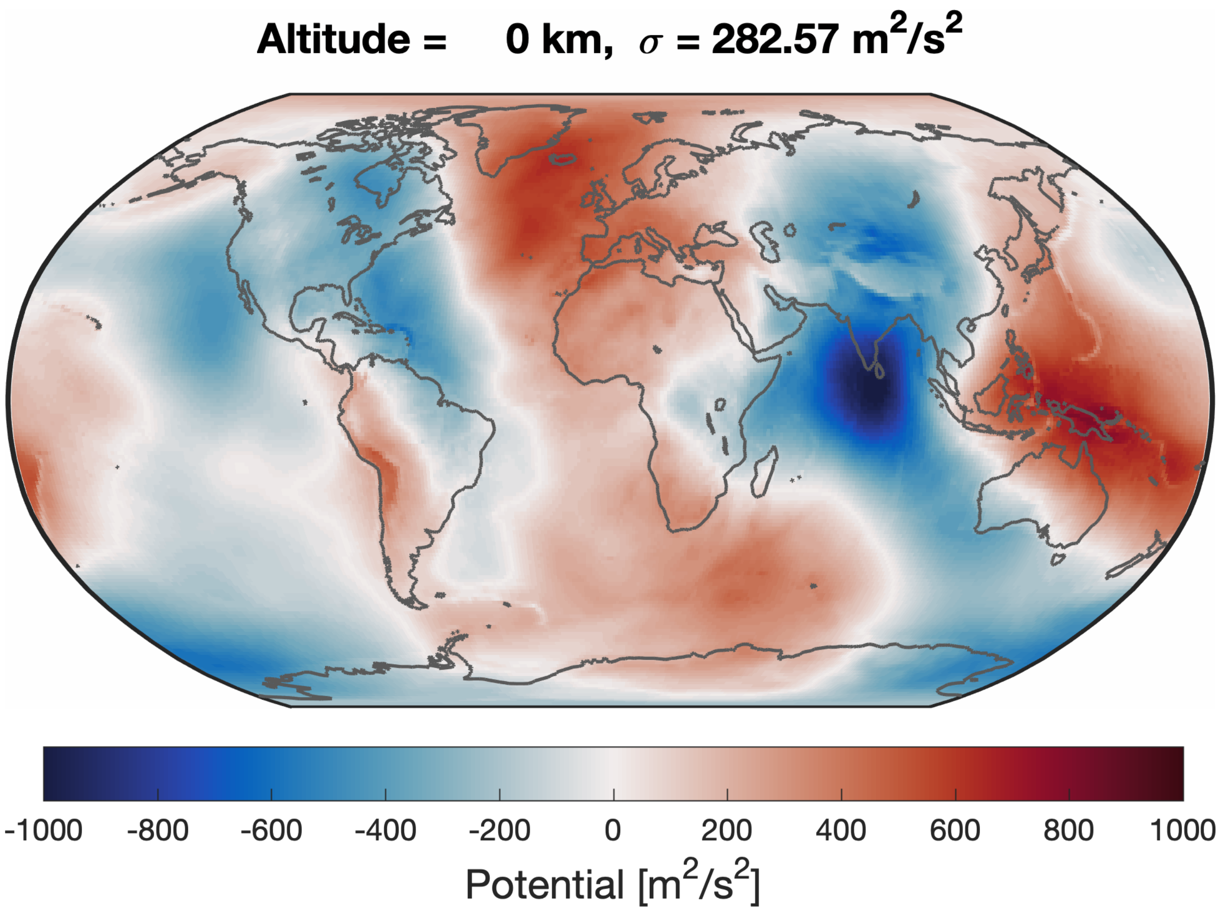
Clocks can detect the mass loss in some areas. Being complementary to GRACE, clocks provide:

- point-wise and
- high-frequency sampling obs.



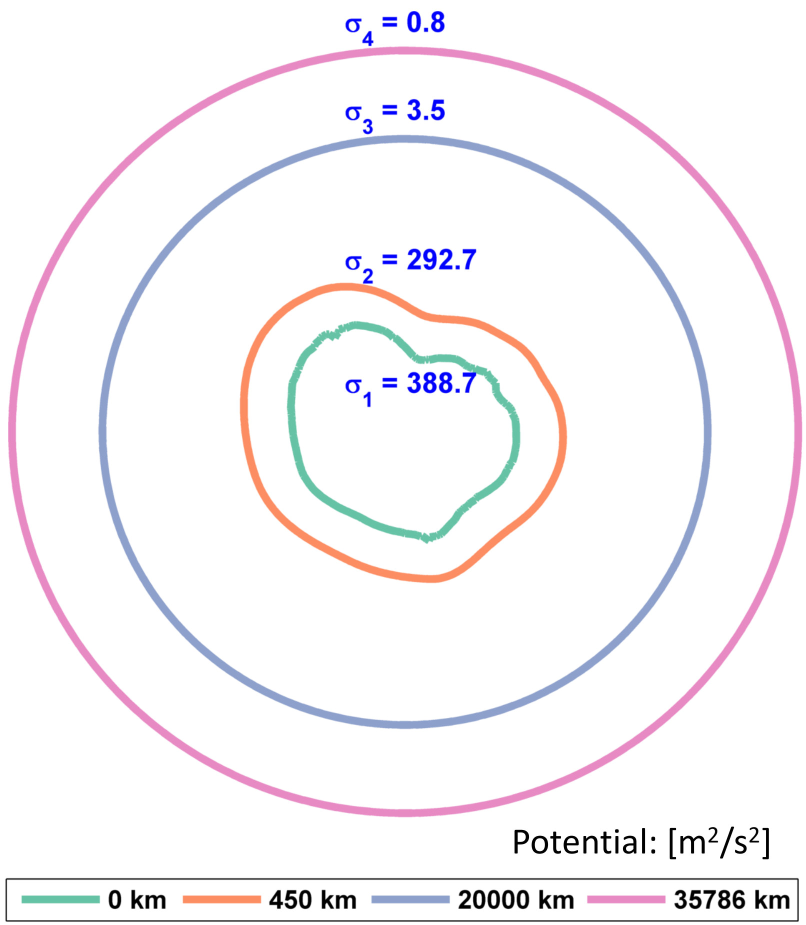
Space geodetic reference frame

Variations of gravity potential at different altitudes

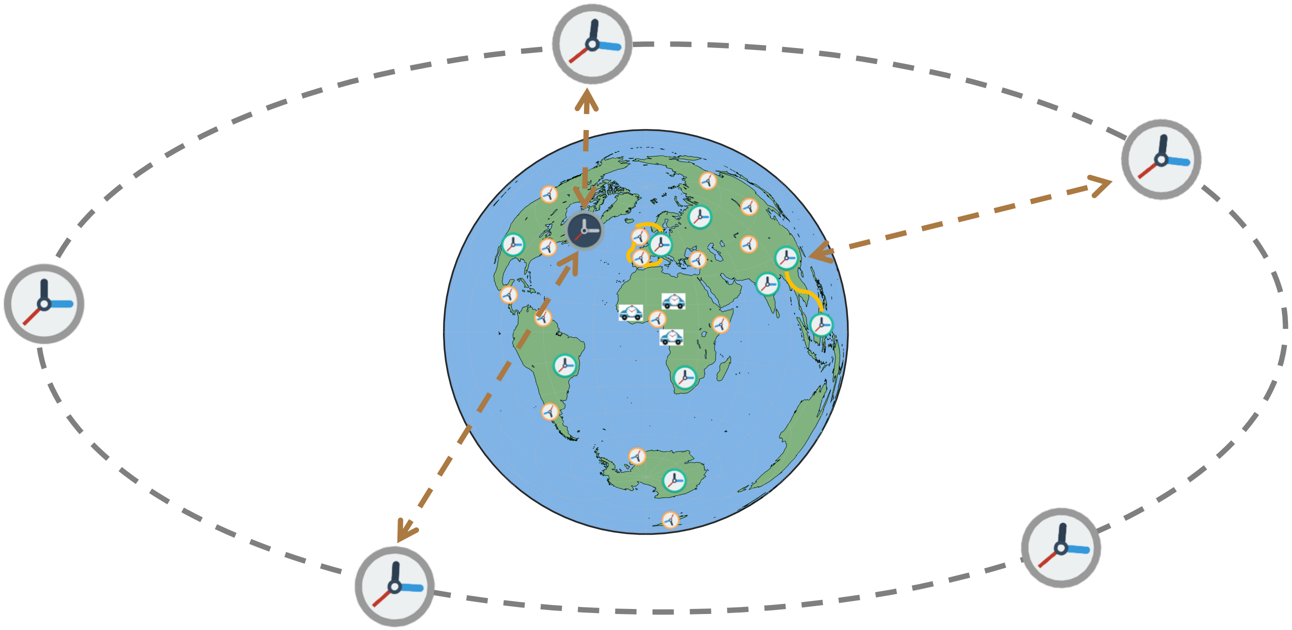


Space geodetic reference frame

Gravity field above the equator at different altitudes



Space geodetic reference frame



Clocks in higher orbits support realizing a global gravity or height reference system, which is:

- stable/robust over time;
- easy to maintain.

Summary

- For gravity field determination, clocks:
  - deliver the gravity potential (difference), which is a scalar quantity and robust to attitude errors;
  - are sensitive to low-degree gravity field signals;
  - can detect the temporal signal below d/o 12 if uncertainty $<10^{-18}$
- As further geodetic applications, clocks can:
  - unify local height systems;
  - monitor mass changes like in Greenland;
  - realize a global gravity/height reference system;
  - ...

Open issues for future work

Doppler effects, configuration of clock networks, procedure for frequency comparison, long-term stability of clocks, ...